Appendix: For Online Publication

Contents

A Data and Measurement

In this section, we explain how we measure the intangible capital stock of public firm.

We use firm level data on public U.S. firms from Compustat covering the period from 1985 to 2016 to measure firm-level intangible capital stock. Our baseline measure of internally generated intangible capital is the sum of two components: (i) estimated knowledge capital, calculated using research and development expenditure (XRD); and (ii) estimated organizational capital, calculated using selling, general, and administrative expenses (XSGA). The measure is constructed using the perpetual inventory method, which aggregates net investment flows over the life of the firm:

$$\begin{aligned} [\text{Knowledge capital}]: \quad & k^G_{i,t} = (1-\delta_G)\, k^G_{i,t-1} + R\&D_{it}, \\ [\text{Organizational capital}]: \quad & k^O_{i,t} = (1-\delta_O)\, k^O_{i,t-1} + \gamma_O SG\&A_{it}, \end{aligned}$$

where $R\&D$ is research and development expenditure expenditure; $SG\&A$ is selling, general, and administrative expenses. All the intangible flow variables are deflated by the price of intellectual property products from National Income and Product Accounts data (NIPA Table 1.1.9, line 12). δ_G and δ_O are the depreciation rates.[1] γ_O is the fraction of selling,general, and administrative ($SG\&A$) expenditure that adds to the intangible capital stock. We assume $\gamma_O = 0.20$ following Falato et al. (2022). All the empirical results are robust over other reasonable choices of this parameter level.

Then, we calculate the net change in the acquired amount of intangibles from changes in the book values of intangibles after the amortization, using Compustat variables INTAN and AM. We obtain the acquired intangible stock $k^B_{i,t}$, applying the perpetual inventory method to the deflated net change in the intangibles.

[1] We use $\delta_G = \delta_O = 0.15$, which is around the levels estimated in the literature (Corrado, Hulten, and Sichel, 2009).

Our final measure of firm-level intangible capital stock $k^I_{i,t}$ is obtained by combining the internally generated intangible stocks and the acquired intangibles stocks:

$$k^I_{i,t} = k^G_{i,t} + k^O_{i,t} + k^B_{i,t}$$

B Industry Level Analsys and Additional Figures

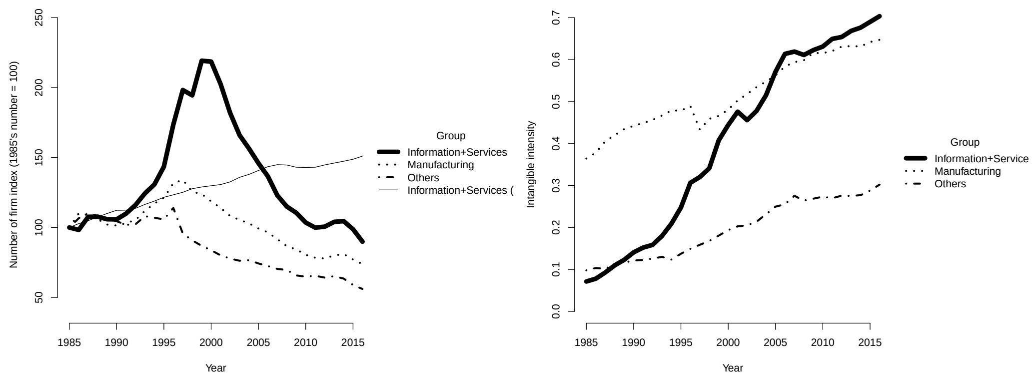
Panel B.1a shows the trend in the number of firms for the Information and Services sector, excluding trade and transportation, Manufacturing, and other sectors (Trade and transportation, Agriculture and Mining, Construction). All sectors show an initial increase and then a decline after the mid-nineties, and the decline is much more pronounced in the information and service sector. We also plot the normalized number of all non-listed firms in the information and service sector: as can be seen, only listed firms are affected by the large decline during the entire period of 2000 and 2010 (well after the dot-com bubble), while the overall number is only slightly affected by the 2001 and 2008 recessions. Panel B.1b shows the *intangible intensity*, defined as the ratio of intangible asset to total intangible and tangible asset values, for the same industries. Manufacturing had historically a higher intangible intensity, which has been taken over in the early 2000s by the service sector.

Finally, Figure B.2 shows our transparency measures for the information and service sector, compared to all other sectors. The information and service sector has a lower transparency over the entire period, and both time series of transparency for all sectors have also declined over time.

Two main take-aways can be taken by the analysis of industry trends: the information and services sector has seen the largest increase in its intangible intensity, and at the same time the largest decline in the number of listed firms and in transparency.

We finally show the trends for more disaggregated industries. We also report the trends in intangible capital using internally generated R&D only, so that the numbers on intangible intensity can be compared to the ones available in the Bureau

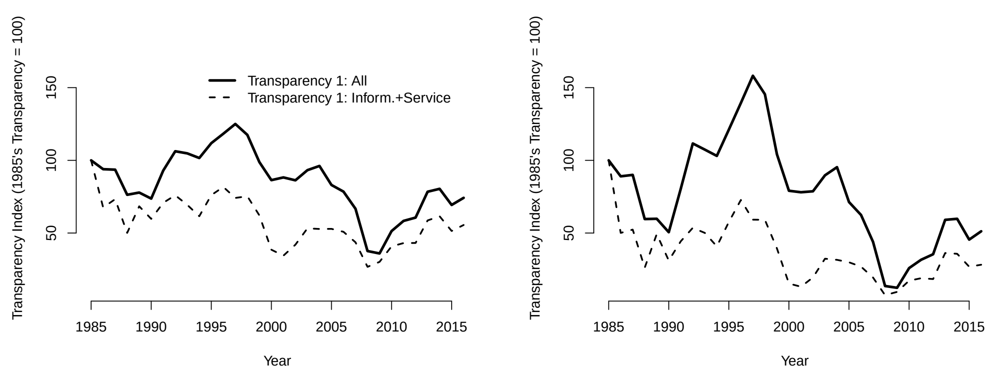
Figure B.1: Number of listed firms and intangible intensity by industry.



(a) Number of listed firms (b) Intangibles intensity

Notes: This figure shows the trend in the number of listed firms and intangible capital intensity in the U.S. Intangible intensity is defined as the ratio of intangible asset to total intangible and tangible asset values. The groups are defined as Information and Services, excluding trade and transportation, Manufacturing,and other sectors (Trade and transportation, Agriculture and Mining, Construction). Data comes from Compustat. See Appendix for details on measurement.

Figure B.2: Time series of transparency for information and service industries.



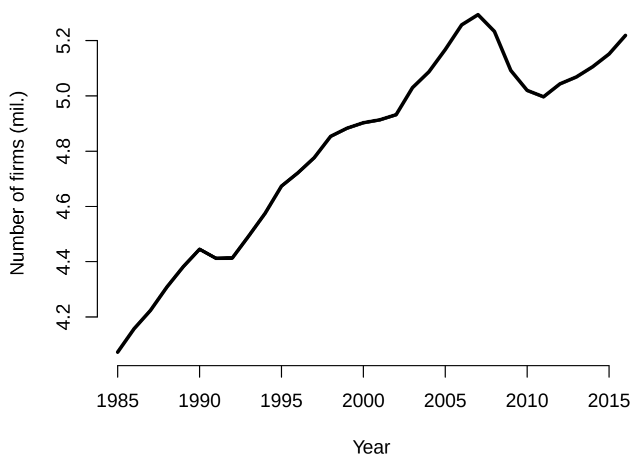
(a) Transparency 1 (b) Transparency 2

Notes: This figure shows the trend in in transparency for information and service industries compared to all other industries. Information and Services excludes trade and transportation. Data comes from Compustat and I/B/E/S. See Appendix for details on measurement.

of Economic Analysis (BEA).

Figure A.1 plots the time series of the total number of firms in the united states. In contrast to the declining trend of the number of listed firms, the total number of firms has been steadily rising.

Figure B.3: Number of all firms

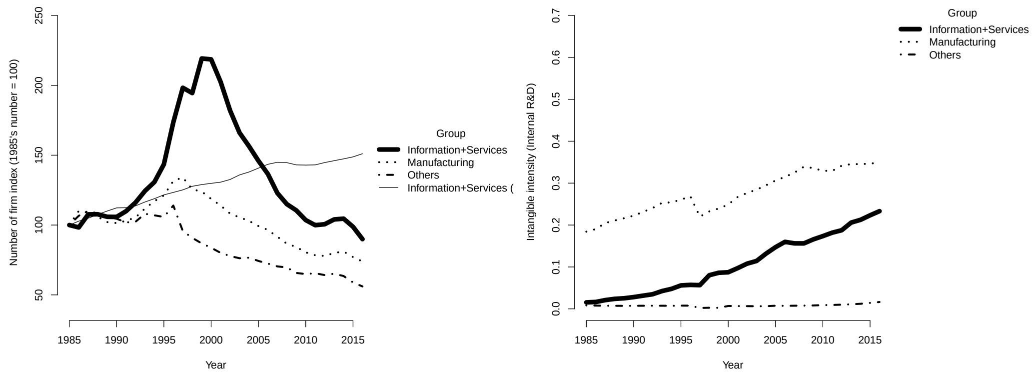


Notes: This figure plots the time series of the number of all firms using the Business Dynamics Statistics (BDS) data from the U.S. Census Bureau.

Figure A.2 plots the time series of (a) the number of listed firms and (b) the intangible intensity measured by using R&D only for multiple industries. Panel (a) shows the industry and service industries have displayed the most dramatic declines in recent years in terms of the number of listed firms. However, the total number of firms in these industries (solid thin line) has steadily risen over the sample period. Panel (b) shows that the information and service industries have shown a fast increase in reliance on intangible assets. However, compared to Figure 2, the difference between the information and service industries and the manufacturing industry is not as drastic.

Figure A.3 plots the time series of the number of listed firms for different industries. Specifically, we report the trends for (a) natural resources and mining, (b)

Figure B.4: Number of listed firms and intangible intensity by industry (internal R&D only).



(a) Number of listed firms

(b) Intangibles intensity

Notes: This figure shows the trend in the number of listed firms and intangible capital intensity in the U.S. Intangible intensity is defined as the ratio of intangible asset, excluding acquired intangible and organizational capital, to total intangible (again excluding acquired intangible and organizational capital) and tangible asset values.The groups are defined as Information and Services, excluding trade and transportation, Manufacturing,and other sectors (Trade and transportation, Agriculture and Mining, Construction). Data comes from Compustat. See Section 2.1 for details on measurement.

construction, (c) manufacturing, trade, (d) transportation, and utilities, (e) information, (f) professional and business services, (e) education and health services, and (f) leisure and hospitality industries.

Figure A.4 plots the time series of the intangible intensity of listed firms for different industries. Specifically, we report the trends for (a) natural resources and mining, (b) construction, (c) manufacturing, trade, (d) transportation, and utilities, (e) information, (f) professional and business services, (e) education and health services, and (f) leisure and hospitality industries.

Figure A.5 plots the time series of the transparency measures for all firms and for only survivors. Here, the survivor is ex-post conditioned by the firms of which observations are available at the end of the sample period.

Figure B.5: Trends in the number of public firms by industry

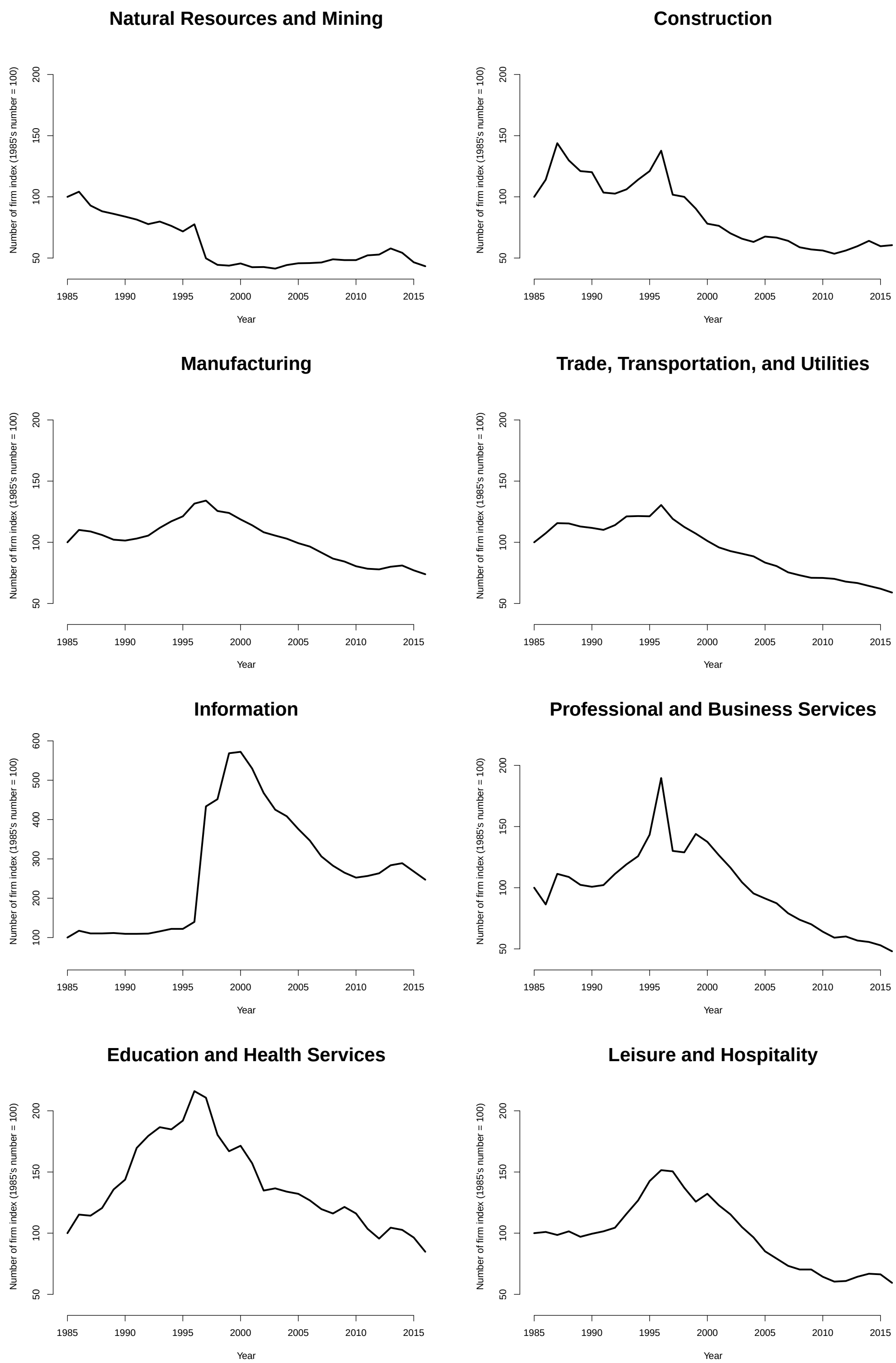


Figure B.6: Trends in intangible intensity by industry

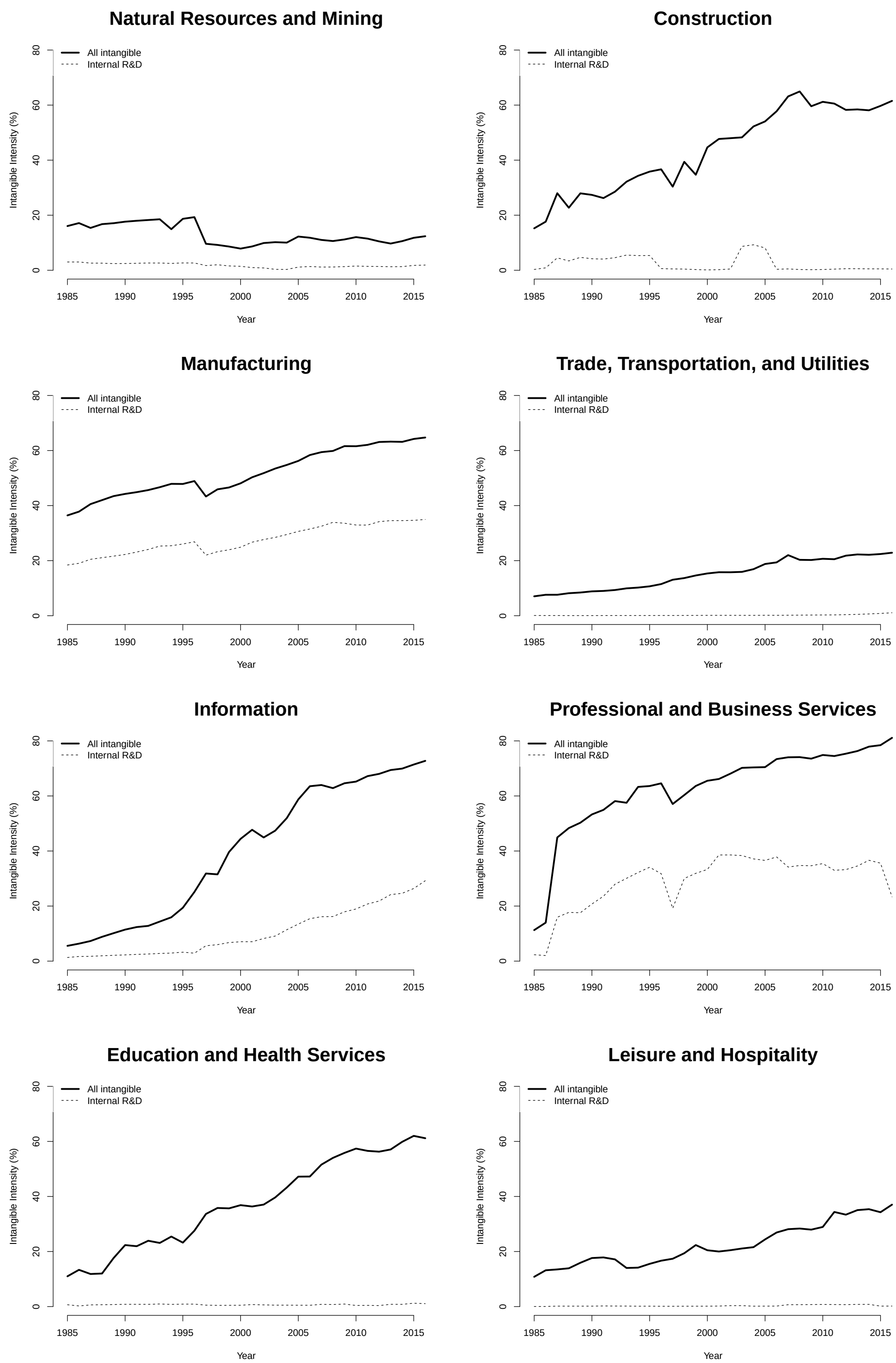
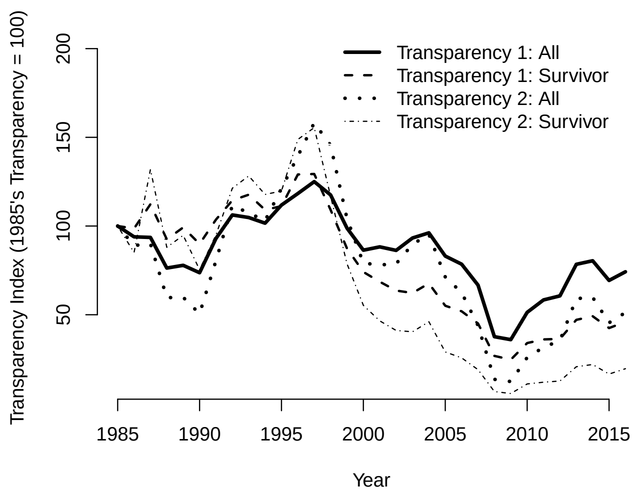


Figure B.7: Time series of transparency: all firms vs. survivors



Notes: This figure shows the trend in in transparency for all firms vs. survivors. Data comes from Compustat and I/B/E/S. See Section 2.1 for details on measurement.

C Cross-sectional evidence: Full table

Table C.1: Regression of transparency proxies on intangibles

	Transparency 1		Transparency 2	
	(1)	(2)	(3)	(4)
Intangible	-.6386	-.3117	-.3191	-.1529
	(-7.33)	(-3.21)	(-7.71)	(-3.07)
Sales	-.2403	-.3077	-.1253	-.148
	(-3.80)	(-4.13)	(-3.93)	(-4.08)
Current Assets	-1.493	-.0664	-.7733	-.0608
	(-5.57)	(-0.26)	(-5.63)	(-0.49)
Leverage	-3.298	-2.029	-1.599	-.9834
	(-9.10)	(-9.12)	(-9.05)	(-8.98)
B/M	-1.993	-1.434	-.9824	-.7053
	(-8.83)	(-8.60)	(-8.66)	(-8.19)
log(Employment)	.5757	.1227	.2909	.0791
	(13.14)	(3.97)	(13.05)	(4.42)
Age	-.0017	-.3345	-8.7e-04	-.1775
	(-0.60)	(-2.05)	(-0.62)	(-2.21)
#Analysts	.0138	.0162	.0201	.0211
	(0.63)	(1.24)	(2.28)	(5.52)
Year FE	✓	✓	✓	✓
Industry FE	✓		✓	
Firm FE		✓		✓
Adj. R^2	0.295	0.649	0.289	0.634
Observations	78878	77944	76959	76014

Notes: This table reports the estimates of the coefficients from the following regression using our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least two (for the first proxy) or one (for the second proxy) analysts is available:

$$\log y_{i,t} = \theta_t + FEs + \beta \times \text{Intangible capital over total assets}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

where $y_{i,t}$ is either the inverse of variance of earning surprises when more than one analyst forecast is present, or the inverse absolute value of earning surprises from the consensus. θ_t are year fixed effects and FEs include either industry or firm fixed effects. $X_{i,t}$ represents firm controls. Standard errors are clustered at the industry and year level.

Table C.2: Regression of forecast accuracy on intangibles

	Earnings Surprises (Absolute Value)	
	(1)	(2)
Intangible	.3191	.1529
	(7.71)	(3.07)
Sales	.1253	.148
	(3.93)	(4.08)
Current Assets	.7733	.0608
	(5.63)	(0.49)
Leverage	1.599	.9834
	(9.05)	(8.98)
B/M	.9824	.7053
	(8.66)	(8.19)
log(Employment)	-.2909	-.0791
	(-13.05)	(-4.42)
Age	8.7e-04	.1775
	(0.62)	(2.21)
#Analysts	-.0201	-.0211
	(-2.28)	(-5.52)
Year FE	✓	✓
Industry FE	✓	
Firm FE		✓
Adj. R^2	0.289	0.634
Observations	76959	76014

Notes: This table reports the estimates of the coefficients from the following regression using our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least one analyst is available:

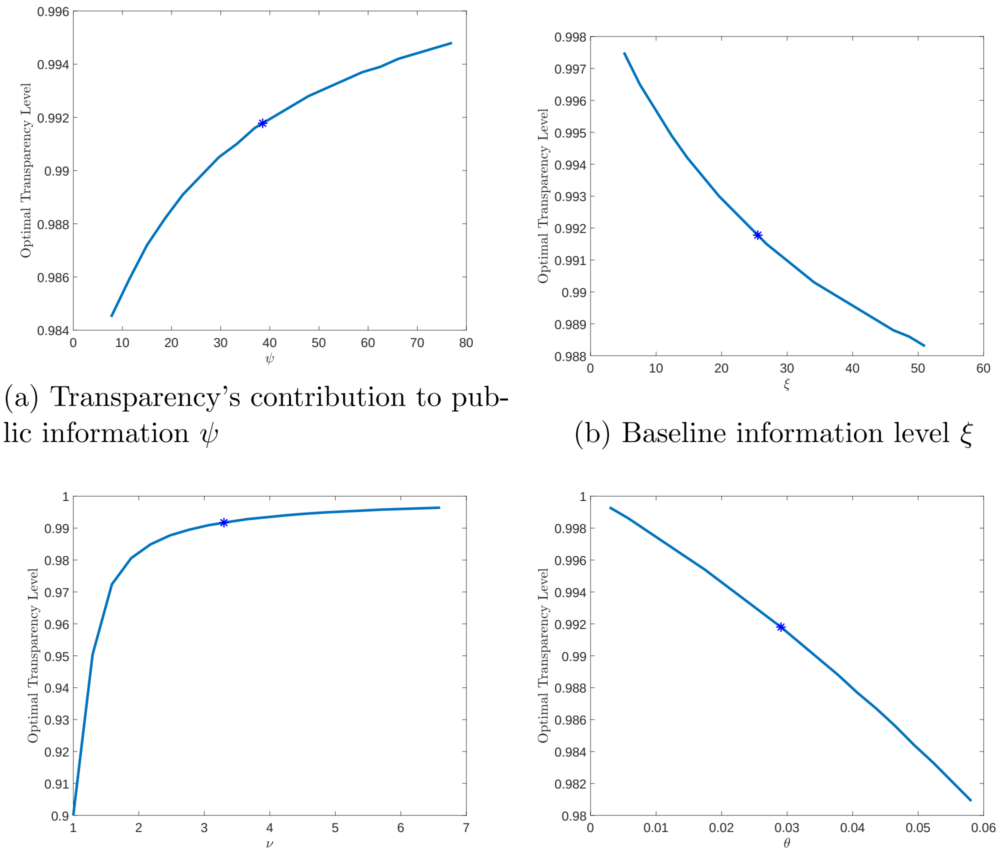
$$\log y_{i,t} = \theta_t + FEs + \beta \times \text{Intangible capital over total assets}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

where $y_{i,t}$ is the absolute value of earning surprises. θ_t are year fixed effects and FEs either industry or firm fixed effects. $X_{i,t}$ represents firm controls. Standard errors are clustered at the industry and year level.

D Comparative Statics

Figure C.6 plots the comparative static results of optimal disclosure regulation (vertical axis) for different parameters (horizontal axis). The optimal regulation displays a strict monotone variation with each parameter change.

Figure D.8: Comparative statics on optimal transparency level with respect to each parameter



(a) Transparency's contribution to public information ψ

(b) Baseline information level ξ

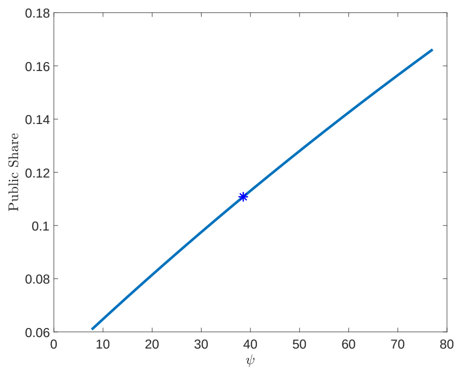
(c) PE market friction ν_N

(d) Intangible share θ

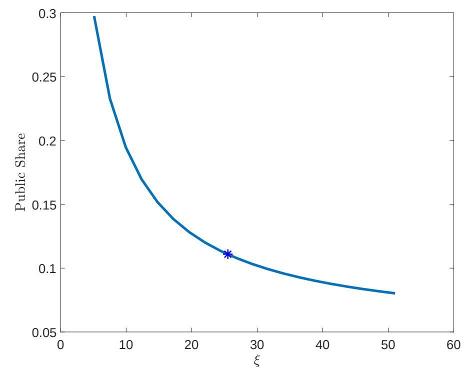
Notes: We change each single parameter, keeping the others constant at their baseline value, and calculate the resulting optimal transparency level.

Figure C.7 plots the comparative static results of the equilibrium mass of listed firms (vertical axis) for different parameters (horizontal axis). The equilibrium mass of listed firms displays a strict monotone variation with each parameter change.

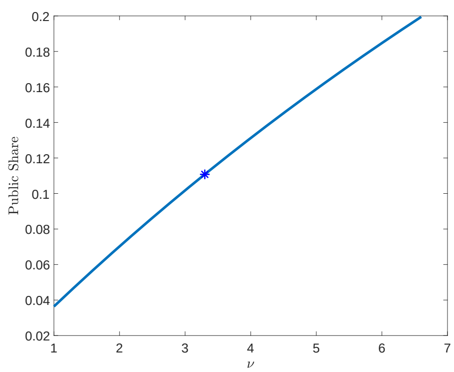
Figure D.9: Comparative statics on fraction of listed firms with respect to each parameter



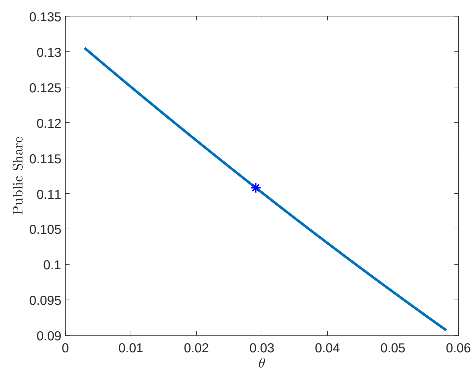
(a) Transparency's contribution to public information ψ



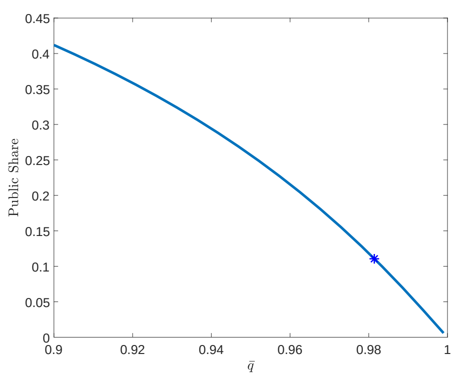
(b) Baseline information level ξ



(c) PE market friction ν_N



(d) Intangible share θ



(e) Mandated minimum transparency $\bar{q}$

Notes: We change each single parameter, keeping the others constant at their baseline value, and calculate the resulting number of listed firms.

E Summary of a firm's problem

A firm's manager decides whether to go listed or non-listed before the operation. If a firm becomes non-listed, the manager does not have to worry about the leakage of their intangibles through disclosure. However, investors penalize the opacity of the non-listed firms by allowing only a low funding intensity.

If a firm becomes public, the manager should decide on the level of transparency $q \geq 0$. If too many firms choose the same transparency level, it will decrease the firm's value in the listed market due to demand-side competition.

A firm's problem could be summarized as follows:

$$
\begin{aligned}
\text{[Entry decision]} \quad & \max\{J^L(\mathcal{M}), J^N(M_N)\}, \\
\text{[Listed firm]} \quad & J^L(\mathcal{M}) := \max_q \max_{k_T, k_I} \left(z k_T^\alpha (k_I(1-\overline{q}-q))^\theta (\Phi^{ex})^\gamma - r k_T - p k_I \right) \phi^L(q) \\
& \quad \text{s.t. } \phi^L(q) = \frac{\xi + \psi(\overline{q}+q)}{\mathcal{M}(q)}, \\
\text{[Non-listed firm]} \quad & J^N(M_N) := \max_{k_T, k_I} \left(z k_T^\alpha (k_I)^\theta (\Phi^{ex})^\gamma - r k_T - p k_I \right) \phi^N \\
& \quad \text{s.t. } \phi^N := \xi / (\nu_N M_N).
\end{aligned}
$$

F The scoreboards: Welfare, productivity, and output

In this section, we define the three objectives of the disclosure regulation: welfare, productivity, and output. First, we define the welfare measure. Besides the performance of firms, the investor values the transparency of the disclosed information, as it is helpful for the investor's portfolio. The representative investor's utility can be monotonically transformed into the following mean-variance form:

$$
\begin{aligned}
&Objective_{welfare} = \int x(\widetilde{q})\frac{\pi(\widetilde{q})}{p(\widetilde{q})}d\widetilde{q} + x^N\frac{\pi^N}{P^N} - \frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}(x^N)^2\frac{1}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{\Lambda}{2}\int \frac{x(\widetilde{q})P(q)\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}\frac{x^N\nu_N P^N M_N}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{\Lambda}{2}\int \frac{\frac{\pi(q)/P(q)}{\Lambda/(\xi+\psi(\overline{q}+q))}P(q)\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}\frac{\frac{\pi^N/P^N}{\Lambda/\xi}\nu_N P^N M_N}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{1}{2}\int \pi(\widetilde{q})\mathcal{M}(\widetilde{q})d\widetilde{q} - \frac{1}{2}\nu_N\pi^N M_N. \\
&= \frac{1}{2}\int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \frac{\nu_N}{2}M^N\pi^N. \qquad (1)
\end{aligned}
$$

Therefore, the welfare measure is equivalent to the expected profit in equilibrium.

The second measure is the productivity in the production sector that is defined as follows:

$$
\begin{aligned}
Objective_{productivity} &= (\Phi^{ex})^\gamma \\
&= \left(\int_0^{1-\overline{q}} (\overline{q}+q)k_I(q,\mathcal{M};\overline{q})\mathcal{M}(q)dq\right)^\gamma,
\end{aligned}
$$

where $\widetilde{A} := \left(\left(\frac{\alpha z}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)$. The productivity is identical to the externality effect, which is the total shared knowledge in the economy. From the regulator's perspective, there is a trade-off in the productivity measure for increasing the strictness of the disclosure requirement. For higher $\overline{q}$, the amount of shared information is greater, while the pool of listed firms to share the information shrinks due to the firm-level extensive-margin responses. Also, the size of intangible k_I to be shared declines as in Proposition 2.

The third measure is the aggregate output in the economy. The output measure is defined in the following form:

$$Objective_{output} = \int_0^{1-\overline{q}} zk_T(q)^{\alpha}(k_I(q)(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}M(q) + zk_{DT}^{\alpha}k_{DI}^{\theta}(\Phi^{ex})^{\gamma}M_N.$$

In our structural structural, we analyze the macroeconomic impact of rising intangible shares and the disclosure policy change, using these three macro-level measures.

G Proofs

G.1 Proof for Proposition 1.

Proposition 1 (Funding supply)**.**

The household's optimal funding supplies for listed firms with transparency q, $x^*(q)$, and for non-listed firms, x^{N*} are as follows:

$$x^*(q) = \frac{\pi(q)/P(q) - \mu}{\Lambda/(\xi + \psi(\overline{q} + q))}$$
$$x^{N*} = \frac{\pi^N/P^N - \mu}{\Lambda/\xi}.$$

Proof.

From the $i.i.d$ assumption of the stock return uncertainty, the consumption (income) satisfies

$$C \sim N\left(\int x(\widetilde{q})\overline{r}(\widetilde{q})d\widetilde{q} + x^N\overline{r}^N, \int x(\widetilde{q})^2 \frac{1}{\xi + \psi(\overline{q} + q)} d\widetilde{q} + (x^N)^2 \frac{1}{\xi}\right).$$

Then the investors' expected utility maximization problem is translated into the following form:[2]

$$\max_{\int x(\widetilde{q})d\widetilde{q} + x^N = a} \quad - e^{-\Lambda\left(\int x(\widetilde{q})\frac{\pi(\widetilde{q})}{P(\widetilde{q})}d\widetilde{q} + x^N\frac{\pi^N}{P^N} - \frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}(x^N)^2\frac{1}{\xi}\right)}.$$

After a strictly-increasing (log) transformation, the problem reduces down to

$$\max_{\int x(\widetilde{q})d\widetilde{q} + x^N = a} \quad \int x(\widetilde{q})\frac{\pi(\widetilde{q})}{P(\widetilde{q})}d\widetilde{q} + x^N\frac{\pi^N}{P^N} - \frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi + \psi(\overline{q} + q)}d\widetilde{q} - \frac{\Lambda}{2}(x^N)^2\frac{1}{\xi}.$$

The first-order condition with respect to $x(q)$ yields

[2] The derivation of the mean-variance portfolio objective function is as follows: consider a random variable, $y \sim N(\mu_y, \sigma_y^2)$. Then,

$$\mathbb{E}(-e^{-\Lambda y}) = -\mathbb{E}(e^{-\Lambda y}) = -e^{-\Lambda(\mu_y - \frac{\Lambda}{2}\sigma_y^2)}.$$

The last equation is immediate from the moment generating function of the normal distribution.

$$\frac{\pi(q)}{P(q)} - \Lambda x^*(q)\frac{1}{\xi + \psi(\overline{q} + q)} - \mu = 0,$$

where μ is the Lagrange multiplier of the wealth constraint. From this equation, we can derive the following supply curve of funding for the listed market:

$$x^*(q) = \frac{\pi(q)/P(q) - \mu}{\Lambda/(\xi + \psi(\overline{q} + q))},$$

where $x^*(q)$ is the funding supply in a dollar amount for firms with the transparency level q. So, the household is willing to invest $\frac{\pi(q)/P(q)-\mu}{\Lambda\frac{1}{\xi+\psi(\overline{q}+q)}}$ in the firms with transparency level q. As we assume the representative household has a large enough wealth a, $\mu = 0$.

Similarly, from the first-order condition with respect to x^N, the funding supply curve for non-listed firms is characterized as follows:

$$x^{N*} = \frac{\pi^N/P^N}{\Lambda/\xi}.$$

■

G.2 Proof for Proposition 2.

Proposition 2. *(Intangibles and the transparency)*
Given $\alpha + \theta < 1$, $k^I(q, \mathcal{M}; \overline{q})$ decreases in both q and $\overline{q}$. Specifically,

$$k_I(q, \mathcal{M}; \overline{q}) = \left(\left(\frac{\alpha z(\Phi^{ex})^\gamma}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)(1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}}.$$

Proof.
From FOC

$$\begin{aligned}
[k_T]: &\quad z\alpha k_T^{\alpha-1}(k_I(1-\overline{q}-q))^\theta(\Phi^{ex})^\gamma = r \\
[k_I]: &\quad z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^\gamma(1-\overline{q}-q) = p \\
&\quad + \left(zk_T^{\alpha}(k_I(1-\overline{q}-q))^\theta(\Phi^{ex})^\gamma - rk_T - pk_I\right)\phi'^L(q) = 0.
\end{aligned}$$

From the first-order conditions with respect to k_T and k_I, we obtain

$$\frac{r}{p} = \left(\frac{\alpha}{\theta}\right) \frac{k_I}{k_T}.$$

Substituting this relation into the first-order condition with respect to k_T, we get

$$r = \alpha z \left(\frac{\alpha p}{\theta r}\right)^{\alpha-1} (k_I)^{\alpha+\theta-1}(1-\overline{q}-q)^{\theta}(\Phi^{ex})^{\gamma}.$$

Thus,

$$k_I = \left(\left(\frac{\alpha z (\Phi^{ex})^{\gamma}}{r}\right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right) (1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}} = A(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}},$$

where $A := \left(\left(\frac{\alpha z (\Phi^{ex})^{\gamma}}{r}\right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right)$. As $\alpha+\theta < 1$, the proposition is immediate from the last equation. ■

G.3 Proof for Proposition 3.

Proposition 3. *(Transparency distribution)*
The probability density function $\mathcal{M}$ of transparency q has the following closed form:

$$\mathcal{M}(q) = (\xi + \psi(\overline{q}+q))\,(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}} \frac{1}{\phi^N}.$$

Proof.

We derive the following equations using the first-order condition with respect to q:[3]

$$\begin{aligned}\frac{\phi'^{L}(q)}{\phi^{L}(q)} &= \frac{z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}k_I}{zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}-rk_T-pk_I} \\ &= \frac{z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}k_I}{(1-\alpha-\theta)zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}} \\ &= \frac{\theta}{1-\alpha-\theta}\left(\frac{1}{1-\overline{q}-q}\right).\end{aligned}$$

From $\frac{\partial}{\partial q}log(\phi^{L}(q)) = \frac{\phi'^{L}(q)}{\phi^{L}(q)}$, the solution of the first-order differential equation is as follows:

$$\phi^{L}(q) = (1-\overline{q}-q)^{n}\,\widetilde{C},$$

for some $n \in \mathbb{R}$ and some $\widetilde{C} \in \mathbb{R}$. From the indifference condition in the equilibrium, $\pi^{L}(q)\phi^{L}(q)$ does not depend on q.

$$\pi^{L}\phi^{L}(q) = \left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\Phi^{\gamma}\right)(1-\overline{q}-q)^{n}\,\widetilde{C}.$$

Therefore,

$$n = -\frac{\theta}{1-\alpha-\theta}.$$

This leads to $\phi^{L}(q) = (1-\overline{q}-q)^{-\frac{\theta}{1-\alpha-\theta}}\,\widetilde{C}$.

Then, the distribution of listed firms is as follows:

$$\begin{aligned}\mathcal{M}(q) &= (\xi+\psi(\overline{q}+q))/\phi^{L}(q) \\ &= (\xi+\psi(\overline{q}+q))\,(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\widetilde{C}}.\end{aligned}$$

[3]Here the proof is based on the first-order conditions that are simultaneously obtained for k_T, k_I, and q for the brevity of notations. The equilibrium allocations stay unaffected in this problem even if the solution is solved sequentially (interim problem first (k_T and k_I), and then q)

From the indifference condition between listed and non-listed,

$$\begin{aligned}\phi^N &= \frac{\pi^L(q)\phi^L(q)}{\pi^N} \\ &= \frac{\left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\Phi^{\gamma}\right)(1-\overline{q}-q)^{-\frac{\theta}{1-\alpha-\theta}}\widetilde{C}}{\left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}\Phi^{\gamma}\right)} \\ &= \widetilde{C}.\end{aligned}$$

Therefore, $\mathcal{M}(q) = (\xi + \psi(\overline{q}+q))(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}} \frac{1}{\widetilde{\phi^N}}$.

In the equilibrium, $\phi^N(=\widetilde{C})$ is determined at the level where the following equation holds:

$$\int_0^{1-\overline{q}} \mathcal{M}(q)dq = 1 - M_N.$$

■

G.4 Proof for Corollary 2.

Corollary 1. *(Truncated Beta distribution)*
The gross transparency, $y := q + \overline{q}$, follows a truncated Beta distribution where the shape parameters are 2 and $B+1$, and the support is $[\overline{q}, 1]$.

$$q + \overline{q} \sim \frac{\mathbb{I}\{q \in [0, 1-\overline{q}]\}}{1 - M_N} \times Beta\,(B+1, 2)\,,$$

where $B = \frac{\theta}{1-\alpha-\theta}$.

Proof.
We define $M_Y(y)$ as the probability density function of the random variable $y = \frac{1-q-\overline{q}}{1+\xi/\psi}$.

$$M_Y(y) \propto (1-y)y^B \quad \text{and} \quad y \in \left[0, \frac{1-\overline{q}}{1+\xi/\psi}\right].$$

Also, $\int_0^{\frac{1-\overline{q}}{1+\xi/\psi}} M_Y(y)dy = 1 - M_N$. Therefore, $y \sim \frac{\mathbb{I}\{q\in[0,1-\overline{q}]\}}{1-M_N} \times Beta\,(B+1, 2)$. ■

G.5 Proof for Proposition 4

Proposition 4 (Non-listed firms' measure)**.**
In equilibrium, the measure of non-listed firms M_N is as follows:

$$M_N = \frac{1}{1 + \psi \frac{\nu_N}{\xi}(1 + \frac{\xi}{\psi})^{B+2} \mathcal{B}(B+1, 2) F\left(\frac{1-\overline{q}}{1+\xi}; B+1, 2\right)}$$

where $\mathcal{B}$ is the beta function, and F is the cumulative distribution function of beta distribution.[4]

Proof. We have the following closed-form solution for M_N:

$$M_N = \frac{1}{1 + \psi \frac{\nu_N}{\xi} \int_0^{1-\overline{q}} (\frac{\xi}{\psi} + (\overline{q} + q))(1 - \overline{q} - q)^B dq}. \tag{2}$$

Using Corollary 1, we can integrate out the $M(q)$ in the right-hand side of the equation in the following steps, using $y = \frac{1-q-\overline{q}}{1+\xi/\psi} \in [0, \frac{1-\overline{q}}{1+\xi/\psi}]$:

$$M_N = \frac{1}{1 + \psi \frac{\nu_N}{\xi}(1 + \frac{\xi}{\psi})^{B+2} \int_0^{\frac{1-\overline{q}}{1+\xi/\psi}} (1-y)(y)^B dy}.$$

$$M_N = \frac{1/\mathcal{B}(B+1, 2)}{1/\mathcal{B}(B+1, 2) + \psi \frac{\nu_N}{\xi}(1 + \frac{\xi}{\psi})^{B+2} \mathcal{B}(B+1, 2) \int_0^{\frac{1-\overline{q}}{1+\xi/\psi}} y^B (1-y) dy}.$$

We integrate the denominator using the cumulative distribution function of beta

[4] The beta function is defined as follows:

$$\mathcal{B}(a, b) := \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a-1)!(b-1)!}{(a+b-1)!} = \int_0^1 x^{a-1}(1-x)^{b-1} dx.$$

distribution, F:

$$M_N = \frac{1/\mathcal{B}(B+1,2)}{1/\mathcal{B}(B+1,2) + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}F\left(\frac{1-\overline{q}}{1+\xi};B+1,2\right)}.$$

By multiplying $\mathcal{B}(B+1,2)$ on the numerator and the denominator, we obtain the following analytic form:

$$M_N = \frac{1}{1 + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2)F\left(\frac{1-\overline{q}}{1+\xi};B+1,2\right)}. \tag{3}$$

■

G.6 Proof for Proposition 5.

Proposition 5. *(The relationship between disclosure regulation and the measure of listed firms)*

M_N *strictly increases in* $\overline{q} \in (0,1)$.

Proof.

We have

$$M_N = \frac{1}{1 + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2)F\left(\frac{1-\overline{q}}{1+\xi};B+1,2\right)}.$$

F decreases in $\overline{q}$, and M_N decreases in F. Thus, M_N increases in $\overline{q}$.

■

References

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